ISSN 1063-7788, Physics of Atomic Nuclei, 2006, Vol. 69, No. 5, pp. 792–795. © Pleiades Publishing, Inc., 2006.
Original Russian Text © A.A. Rukhadze, L.I. Urutskoev, D.V. Filippov, 2006, published in Yadernaya Fizika, 2006, Vol. 69, No. 5, pp. 820–823.

NUCLEI
Theory

Effect of Beta Decay to Bound States in Ionized Atoms on the Fraction of Delayed Neutrons

A. A. Rukhadze, L. I. Urutskoev, and D. V. Filippov

Institute of General Physics, Russian Academy of Sciences, ul. Vavilova 38, Moscow, 119991 Russia

Received May 13, 2005

Abstract—The ionization of an atom leads to the appearance of an additional beta-decay channel to a bound state of an electron. It is shown that, for nuclei that are products of uranium fission and which are emitters of delayed neutrons, the fraction of delayed neutrons increases upon taking into account the additional beta-decay channel to bound states.

PACS numbers : 23.40.-s, 23.90.+w
DOI: 10.1134/S1063778806050024

A theory of β^- decay to a bound electron state, in which case the beta electron does not leave the atom involved, occupying the free orbit, was constructed in [1–4]. The ratios of the decay constants (ratios of the β^--decay probabilities) for transitions to bound and free states (λ_b and λ_c, respectively) were calculated in [3, 4]. For β^- decays of low energy in fully ionized heavy atoms, the ratio λ_b/λ_c may be as large as 10^3–10^4. Thus, we see that, in the presence of free electron orbits, the probability of the β^- decay of nuclei may increase by three orders of magnitude or more. The theory of β^- decay to a bound state was experimentally confirmed in [5, 6].

However, allowance for beta decay to bound states may prove to be of importance not only in the case where the nucleus involved has an anomalously low boundary energy for beta decay (^{187}Re, for example, possesses this property [6]), but also in the case where the decay process being considered proceeds via various channels, including those of decay to highly excited levels of the daughter nucleus. In the first case, the decay half-life changes, while, in the second case, the change in the decay half-life is small, but the ratio of the intensities of decays through different channels may undergo a substantial redistribution. This effect will lead to a change in the relationship between the intensities of the lines of gamma radiation from the daughter nucleus. If, in addition, an emitter of delayed neutrons appears as the initial beta-decaying nucleus, the delayed-neutron fraction will change.

The ratio of the probabilities of β^- decay to bound and free states can be calculated by a method similar to the classic method for calculating the ratio of the K-capture to the β^+-decay probability [7]. In the following, we will use the system of units in which $\hbar = c = m_e = 1$. For allowed beta decays, the decay constants for transitions to a bound and a free state are proportional to the same matrix elements and differ only in phase spaces of electron–neutrino final states.

From [4, 7], it is well known that, for beta decay to a free state, the phase space is proportional to the integral Fermi function

$$f(Z,E) = \int_1^E F(Z,\varepsilon)\varepsilon\sqrt{\varepsilon^2-1}(E-\varepsilon)^2 d\varepsilon \qquad (1)$$

(E is the beta-transition energy) and is the sum of all possible energies and momentum directions for the electron (neutrino). The Fermi function in (1) grows with energy faster than in proportion to E^2.

For beta decay to a bound state, the neutrino spectrum is concentrated at a single energy value, since the energy of an electron moving along an orbit is fixed, and the phase space is determined by a possible arbitrary direction of the neutrino momentum. The phase space is then proportional to the product of the square of the neutrino momentum,

$$p^2 = \frac{1}{c^2}(E-1+\varepsilon)^2 \qquad (2)$$

(ε is the energy of the electron on the respective orbit), and the probability of the intersection of the free electron orbit and the nucleus involved. In turn, the intersection probability is proportional to $|\Psi_e(R)|^2$, where $\Psi_e(R)$ is the density of the electron wave functions in the region of the nucleus.

It follows that the appearance of a free electron orbit enhances the decay constant for the allowed beta

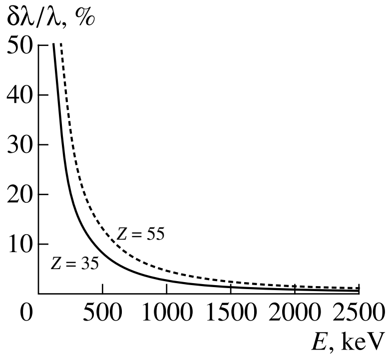

Fig. 1. Ratio of the probability of decay to a bound electron state, $\delta\lambda$, to the probability of decay to a free electron state, λ, as a function of the beta-decay energy E for nuclei of charge number $Z = 35$ and 55.

transition of energy E by the quantity $\delta\lambda$,

$$\frac{\delta\lambda}{\lambda} = 2\pi^2 \frac{|\Psi_e(R)|^2 (E - 1 + \varepsilon)^2}{f(Z, E)}. \qquad (3)$$

Since, with increasing energy E, the Fermi function (1) grows faster than in proportion to E^2, the ratio $\delta\lambda/\lambda$ decreases as the energy increases,

$$\frac{\delta\lambda}{\lambda} \propto \frac{1}{E^{\alpha}}, \quad \alpha > 0. \qquad (4)$$

Thus, the lower the decay energy E, the more pronounced the increase in the decay constant because of decay to a bound state. It is noteworthy that the energy dependence of the decay constant takes the same form, irrespective of which electron orbit is free, since it is the factor $|\Psi_e(R)|^2$ that absorbs the effect of the distinction between the orbits. In order to derive the estimate in (4), we only employed the fact that neutrinos accompanying beta decay to a bound state are monoenergetic.

In the particular case where a hydrogen-like orbit of the atom being considered is free (the case of a fully ionized atom), we find from (3) that

$$\frac{\delta\lambda}{\lambda} \sim \frac{2\pi(\alpha Z)^3 (E - 1 + \varepsilon)^2}{f(Z, E)}, \qquad (5)$$

where $\alpha = 1/137$ is the fine-structure constant and Z is the charge number of the nucleus. In Fig. 1, the ratio $\delta\lambda/\lambda$ is shown as a function of the beta-decay energy at $Z = 35$ and 55. We note that the ratio $\delta\lambda/\lambda$ is greater for forbidden than for allowed transitions since, in expression (5), the numerator features the beta-decay form factor at the maximum neutrino energy, while the denominator is equal to the same form factor averaged over all neutrino energies according to (1). For uniquely forbidden transitions, the ratio $\delta\lambda/\lambda$ was considered in [4].

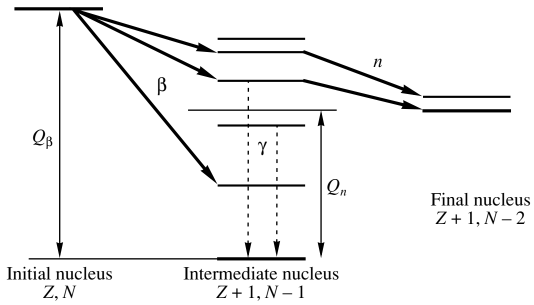

Fig. 2. Scheme of the decay of a nucleus emitting delayed neutrons (Q_β is the maximum beta-decay energy, while Q_n is the neutron binding energy in the intermediate nucleus).

The fission of ^{235}U leads to the formation of a large number of fragments whose atomic weights range between $A = 72$ and $A = 160$. The mass and charge distributions of fission fragments have been well understood. The majority of the fragments are unstable neutron-rich nuclei [8]. Among these, about 50 nuclei are sources of delayed neutrons and decay according to the scheme in Fig. 2.

The beta decay of the initial nucleus (delayed-neutron emitter) through a channel characterized by lower beta-transition energies leads to the formation of an intermediate nucleus in an excited state. At an excitation energy above the neutron binding energy (Q_n), the intermediate nucleus emits a neutron. Neutron emission from the intermediate nucleus is virtually prompt, the delay time being determined by the lifetime of the initial nucleus. As can be seen from Fig. 2, the emergence of delayed neutrons is due to low-energy beta decays [8].

For the majority of intermediate nuclei, the neutron separation energy is $Q_n \sim 4$–7 MeV. The energy of the beta decay that leads to neutron production, $Q_\beta - Q_n$, is substantially lower than Q_β, and one can see from the estimates in (3)–(5) that, in the case where there arise additional channels of decay to a bound state, the ratio $\delta\lambda_n/\lambda_n$ for the neutron channel characterized by low energies E exceeds substantially $\delta\lambda_\beta/\lambda_\beta$ for neutronless decay to low-lying levels:

$$\frac{\delta\lambda_n}{\lambda_n} > \frac{\delta\lambda_\beta}{\lambda_\beta}. \qquad (6)$$

The delayed-neutron fraction β is proportional to the ratio

$$\beta \propto \frac{\lambda_n}{\lambda_n + \lambda_\beta}. \qquad (7)$$

It can readily be found that the relative change in the delayed-neutron fraction is

$$\frac{\delta\beta}{\beta} = \frac{\lambda_\beta}{\lambda} \left(\frac{\delta\lambda_n}{\lambda_n} - \frac{\delta\lambda_\beta}{\lambda_\beta} \right) > 0, \qquad (8)$$

Fission products appearing as emitters of delayed neutrons

Emitting nucleus	Z	A	T, c	β,%	Group	Q_β, MeV	Q_n, MeV	$Q_\beta - Q_n$, MeV	E, keV	$\varepsilon(Z+1)$, keV	ΔE, keV	$\delta\beta/\beta$, %
Br	35	87	55.6	2.52	1	6.83	5.515	1.3	–	17.9	13	2
Cs	55	141	24.9	0.03	2	5.25	4.525	0.7	–	44.6	35	10
I	53	137	24.5	6.97	2	5.88	4.025	1.9	–	41.3	32	2
Te	52	136	17.5	1.1	2	5.09	3.782	1.3	890	39.7	31	4.6
Br	35	88	16.34	6.58	2	8.96	7.053	1.9	660	17.9	13	4.7
I	53	138	6.49	5.5	3	7.82	5.812	2.0	820	41.3	32	5.6
Rb	37	93	5.85	1.38	3	7.462	5.284	2.2	–	20.	15	3
Se	34	87	5.29	0.36	3	7.28	6.289	1.0	980	16.9	12	2.6
As	33	84	4.5	0.28	3	9.9	8.681	1.2	1170	15.9	11	1.9
Rb	37	92	4.492	0.01	3	8.1	7.3	0.8	120	20.	15	∼50
Br	35	89	4.348	13.8	3	8.15	5.104	3.0	1140	17.9	13	1.8

where $\lambda = \lambda_n + \lambda_\beta + \delta\lambda_n + \delta\lambda_\beta$. Therefore, the appearance of a free electron orbit in an atom that emits delayed neutrons leads to an increase in the delayed-neutron fraction.

The table presents data on nuclei that emit neutrons and which originate from the fission of uranium and plutonium from the first three groups [8–10]. In this table, T is the half-life of the initial nucleus, β is the fraction of decays that lead to neutron production, Q_β is the maximum energy of the beta decay of the initial nucleus, Q_n is the neutron binding energy in the intermediate nucleus, and $\delta\beta/\beta$ is the relative increase in the delayed-neutron fraction. The energy E of beta decay that leads to neutron production is indicated only in the cases where a neutron is produced owing to the decay of a single experimentally fixed [10] excited level. In those cases, the increase in the delayed-neutron fraction, $\delta\beta/\beta$, is calculated by formula (8) for a single energy value E (the values of the Fermi function were borrowed from [11]). In the remaining cases, averaging with allowance for the channel-intensity ratios known from [10] is performed in calculating $\delta\beta/\beta$.

In the beta decay of a fully ionized nucleus to a bound electron state, the boundary value of the beta-decay energy increases by ΔE [12, 13],

$$\Delta E = \varepsilon + I(Z) - I(Z+1) \equiv \varepsilon - \Delta I(Z) < \varepsilon, \quad (9)$$

where $I(Z)$ is the total ionization energy of a neutral atom whose charge number is Z, while $\Delta I(Z)$ is the difference of the total ionization energies between the $Z+1$ and Z atoms. In the table, we also quote the energies of electrons in the K shell, ε, of the daughter ion (nucleus of charge number $Z+1$) and the increase in the boundary energy of beta decay, ΔE. For the decay of fully ionized $^{187}Re^{75+}$ [6], we have $\Delta I \sim 15$ keV, $\varepsilon \sim 85$ keV, and $\Delta E \sim 70$ keV, the energy of neutral-atom beta decay being $E = 2.66$ keV. This led to the emergence of decays to higher lying levels of the daughter nucleus. For the majority of nuclei being considered that emit delayed neutrons, ΔE is small in relation to E (at $Z \sim 35$, we have $\Delta I \sim 5$ keV, $\varepsilon \sim 15$ keV, and $\Delta E \sim 10$ keV $\ll E \sim 1$ MeV, while, at $Z \sim 55$, the respective values are $\Delta I \sim 10$ keV, $\varepsilon \sim 40$ keV, and $\Delta E \sim 30$ keV $\ll E$). Despite the smallness of ΔE in relation to E, the increase in the boundary beta-decay energy may lead to the emergence of decays that populate new states from which neutron emission occurs. It can only be said with confidence that this effect will enhance the estimate of $\delta\beta/\beta$ in the table. Unfortunately, highly excited states of intermediate nuclei emitting neutrons have not yet received adequate study. At any rate, we were unable to find experimental information about the existence of levels that could open additional channels of neutron production in the beta decay of an ionized atom to a bound electron state.

Thus, we have seen that the emergence of additional channels of beta decay to bound electron states for nuclei that emit delayed neutrons leads to an increase in the delayed-neutron fraction.

REFERENCES

1. J. N. Bahcall, Phys. Rev. **124**, 495 (1961).
2. K. Takahashi and K. Yokoi, Nucl. Phys. A **404**, 578 (1983).

3. K. Takahashi, R. N. Boyd, G. J. Mathews, and K. Yokoi, Phys. Rev. C **36**, 1522 (1987).
4. I. S. Batkin, Izv. Akad. Nauk SSSR, Ser. Fiz. **40**, 1279 (1976).
5. M. Jung, F. Bosch, K. Beckert, et al., Phys. Rev. Lett. **69**, 2164 (1992).
6. F. Bosch, T. Faestermann, J. Friese, et al., Phys. Rev. Lett. **77**, 5190 (1996).
7. M. A. Preston, *Physics of the Nucleus* (Addison-Wesley, Reading, Mass., 1962; Mir, Moscow, 1964), p. 388.
8. Yu. P. Gangrsky, B. Dalkhsuren, and B. N. Markov, *Nuclear-Fission Fragments* (Énergoatomizdat, Moscow, 1986) [in Russian].
9. V. M. Gorbachev, Yu. S. Zamyatin, and A. A. Lbov, *Interaction of Radiation with Nuclei of Heavy Elements and Nuclear Fission* (Atomizdat, Moscow, 1976) [in Russian].
10. S. Y. F. Chu, L. P. Ekström, and R. B. Firestone, *WWW Table of Radioactive Isotopes* (database version 2/28/1999 from URL), http://nucleardata.nuclear.lu.se/nucleardata/toi/
11. B. S. Dzhelepov, L. N. Zyr'yanov, and Yu. P. Suslov, *Beta Processes* (Nauka, Leningrad, 1972) [in Rusiian].
12. L. I. Urutskoev and D. V. Filippov, Usp. Fiz. Nauk **174**, 1355 (2004) [Phys. Usp. **47**, 1257 (2004)].
13. S. V. Starodubtsev and A. M. Romanov, *Nuclear Transformation and Atomic Shell* (Akad. Nauk UzSSR, Tashkent, 1958), p. 236.

Translated by A. Isaakyan